## EXTRA TOPICS RELATED TO

x5 Internal Op Amp Circuits

## x5.1 The 741 Circuit

## x5.2 Special Low Supply Voltage Performance Requirements

T:
his supplement contains material removed from previous editions of the textbook. These topics continue to be relevant and for this reason will be of great value to many instructors and students.

The topics presented here relate to Chapter 13 of the eighth edition. In particular, the $741 \mathrm{op}-\mathrm{amp}$ circuit is analyzed in full detail. The final section explains special requirements of BJT op amps designed for low supply voltages.

## x5.1 The 741 Circuit

Figure x 5.1 shows the 741 op -amp circuit. In keeping with the IC design philosophy, the circuit uses a large number of transistors, but relatively few resistors and only one capacitor. This philosophy is dictated by the economics (silicon area, ease of fabrication, quality of realizable components) of the fabrication of active and passive components in IC form (see Section 8.1 and Appendix A).

As in the case of most general-purpose IC op amps, the 741 requires two power supplies, $+V_{C C}$ and $-V_{E E}$. Normally, $V_{C C}=V_{E E}=15 \mathrm{~V}$, but the circuit also operates satisfactorily with the power supplies reduced to much lower values (such as $\pm 5 \mathrm{~V}$ ). It is important to observe that no circuit node is connected to ground, the common terminal of the two supplies.

With a relatively large circuit like that shown in Fig. x5.1, the first step in the analysis is to identify its recognizable parts and their functions. Thus, we begin with a qualitative description of the circuit. Our description is aided by the division of the circuit into its various parts, as indicated in the diagram.

Figure x5.1 The 741 op-amp circuit: $Q_{11}, Q_{12}$, and $R_{5}$ generate a reference bias current, $I_{\text {REF }} \cdot Q_{10}, Q_{9}$, and $Q_{8}$ bias the input stage, which is composed of $Q_{1}$ to $Q_{7}$. The and an input buffer $Q_{23}$. Transistors $Q_{15}, Q_{21}, Q_{21}$, and $Q_{22}$ serve to protect the amplifier against output short circuits and are normally cut off.

Bias Circuit The reference bias current of the 741 circuit, $I_{\mathrm{REF}}$, is generated in the branch at the extreme left of Fig. x5.1, consisting of the two diode-connected transistors $Q_{11}$ and $Q_{12}$ and the resistance $R_{5}$. Using a Widlar current source formed by $Q_{11}, Q_{10}$, and $R_{4}$, bias current for the first stage is generated in the collector of $Q_{10}$. Another current mirror formed by $Q_{8}$ and $Q_{9}$ takes part in biasing the first stage.

The reference bias current $I_{\text {REF }}$ is used to provide two proportional currents in the collectors of $Q_{13}$. This double-collector lateral pnp transistor can be thought of as two transistors whose base-emitter junctions are connected in parallel. Thus $Q_{12}$ and $Q_{13}$ form a two-output current mirror: One output, the collector of $Q_{13 B}$, provides bias current and acts as a current-source load for $Q_{17}$, and the other output, the collector of $Q_{13 A}$, provides bias current for the output stage of the op amp.

Two more transistors, $Q_{18}$ and $Q_{19}$, take part in the dc bias process. The purpose of $Q_{18}$ and $Q_{19}$ is to establish two $V_{B E}$ drops between the bases of the output transistors $Q_{14}$ and $Q_{20}$.

Short-Circuit-Protection Circuitry The 741 circuit includes a number of transistors that are normally off and conduct only if one attempts to draw a large current from the opamp output terminal. This happens, for example, if the output terminal is short-circuited to one of the two supplies. The short-circuit-protection network (shown in color in Fig. x5.1) consists of $R_{6}, R_{7}, Q_{15}, Q_{21}, Q_{24}, R_{11}$, and $Q_{22}$. In the following we shall assume that these transistors are off. Operation of the short-circuit-protection network will be explained in Section x5.1.3 of this supplement.

The Input Stage The 741 circuit consists of three stages: an input differential stage, an intermediate single-ended high-gain stage, and an output-buffering stage. The input stage consists of transistors $Q_{1}$ through $Q_{7}$, with biasing performed by $Q_{8}, Q_{9}$, and $Q_{10}$. Transistors $Q_{1}$ and $Q_{2}$ act as emitter followers, causing the input resistance to be high and delivering the differential input signal to the differential common-base amplifier formed by $Q_{3}$ and $Q_{4}$. Thus the input stage is the differential version of the common-collector, common-base configuration discussed in Section x3.3 of the extra topics.

Transistors $Q_{5}, Q_{6}$, and $Q_{7}$ and resistors $R_{1}, R_{2}$, and $R_{3}$ form the load circuit of the input stage. This is an elaborate current-mirror-load circuit, which we will analyze in Section x5.1.3 of this supplement. The circuit is based on the base-current-compensated mirror studied in Section 8.2.4, but it includes two emitter-degeneration resistors $R_{1}$ and $R_{2}$, and a large resistor $R_{3}$ in the emitter of $Q_{7}$. As is the case with current-mirror loads, this circuit not only provides a high-resistance load for $Q_{4}$ but also converts the signal from differential to single-ended form with no loss in gain or common-mode rejection. The output of the input stage is taken single-endedly at the collector of $Q_{4}$.

Every op-amp circuit includes a level shifter whose function is to shift the dc level of the signal so that the signal at the op-amp output can swing positive and negative. In the 741 , level shifting is done in the first stage using the lateral pnp transistors $Q_{3}$ and $Q_{4}$. Although lateral pnp transistors have poor high-frequency performance, their use in the common-base configuration (which is known to have good high-frequency response) does not seriously impair the op-amp frequency response.

The use of the lateral pnp transistors $Q_{3}$ and $Q_{4}$ in the first stage results in an added advantage: protection of the input-stage transistors $Q_{1}$ and $Q_{2}$ against emitter-base junction breakdown. Since the emitter-base junction of an npn transistor breaks down at about 7 V of reverse bias (see Section 6.4.1 of the textbook), regular npn differential stages suffer such a breakdown if, say, the supply voltage is accidentally connected between the input terminals. Lateral pnp transistors, however, have high emitter-base
breakdown voltages (about 50 V ), and because they are connected in series with $Q_{1}$ and $Q_{2}$, they provide protection of the 741 input transistors, $Q_{1}$ and $Q_{2}$.

Finally, note that except for using input buffer transistors, the 741 input stage is essentially a current-mirror-loaded differential amplifier. It is quite similar to the input stage of the CMOS amplifier in Fig. 13.1.

The Second Stage The second or intermediate stage is composed of $Q_{16}, Q_{17}, Q_{13 B}$, and the two resistors $R_{8}$ and $R_{9}$. Transistor $Q_{16}$ acts as an emitter follower, thus giving the second stage a high input resistance. This minimizes the loading on the input stage and avoids loss of gain. Also, adding $Q_{16}$ with its $50-\mathrm{k} \Omega$ emitter resistance (which is similar to $Q_{7}$ and $R_{3}$ ) increases the symmetry of the first stage and thus improves its CMRR. Transistor $Q_{17}$ acts as a common-emitter amplifier with a $100-\Omega$ resistor in the emitter. Its load is composed of the high output resistance of the pnp current source $Q_{13 B}$ in parallel with the input resistance of the output stage (seen looking into the base of $Q_{23}$ ). Using a transistor current source as a load resistance (active load) enables one to obtain high gain without resorting to the use of large resistances, which would occupy a large chip area and require large power-supply voltages.

The output of the second stage is taken at the collector of $Q_{17}$. Capacitor $C_{C}$ is connected in the feedback path of the second stage to provide frequency compensation using the Miller compensation technique studied in Section 11.10.3 of the textbook. In Section x5.1.4 of this supplement we will see that the relatively small capacitor $C_{C}$ gives the 741 a dominant pole at about 4 Hz . Furthermore, pole splitting causes other poles to be shifted to much higher frequencies, giving the op amp a uniform $-20-\mathrm{dB} /$ decade gain rolloff with a unity-gain bandwidth of about 1 MHz . Note that although $C_{C}$ is small in value, the chip area that it occupies is about 13 times that of a standard npn transistor!

THE CREATOR OF THE $\mu A 741$ : DAVID FULLAGAR

David Fullagar was at Fairchild Semiconductor in 1967 when he designed the $\mu \mathrm{A} 741$, perhaps the most successful op amp ever. Fairchild, TI, and National still sell updated versions of this ubiquitous device. Fullagar, educated at Cambridge, U.K., and formerly employed at Ferranti, had joined Fairchild in 1966 following Widlar's departure after the $\mu \mathrm{A} 702$ and $\mu \mathrm{A} 709$ designs. Fullagar's $\mu \mathrm{A} 741$ creation incorporated internal compensation, short-circuit protection, and a novel highimpedance input stage to resolve shortcomings of the earlier designs. After leaving Fairchild, he joined Intersil as the company's first analog IC designer. The engineer-designer cofounded and became a vital technical contributor to Maxim Integrated Products in 1983; he retired in 1999.

The Output Stage The purpose of the output stage (Chapter 12 of the eighth edition) is to provide the amplifier with a low output resistance. In addition, the output stage should be able to supply relatively large load currents without dissipating an unduly large amount of power in the IC. The 741 uses an efficient class AB output stage, which we shall study in Section x5.1.3 of this supplement.

The output stage of the 741 consists of the complementary pair $Q_{14}$ and $Q_{20}$, where $Q_{20}$ is a substrate pnp (see Appendix A). Transistors $Q_{18}$ and $Q_{19}$ are fed by current source $Q_{13 A}$ and bias the output transistors $Q_{14}$ and $Q_{20}$. Transistor $Q_{23}$ (which is another substrate $р п р)$ acts as an emitter follower, thus minimizing the loading effect of the output stage on the second stage.

Device Parameters In the following sections and exercises we carry out a detailed analysis of the 741 circuit. For the standard $n p n$ and $p n p$ transistors, the following parameters will be used:

$$
\begin{array}{ll}
n p n: & I_{S}=10^{-14} \mathrm{~A}, \beta=200, V_{A}=125 \mathrm{~V} \\
\text { pnp: } & I_{S}=10^{-14} \mathrm{~A}, \beta=50, V_{A}=50 \mathrm{~V}
\end{array}
$$

In the 741 circuit the nonstandard devices are $Q_{13}, Q_{14}$, and $Q_{20}$. Transistor $Q_{13}$ will be assumed to be equivalent to two transistors, $Q_{13 A}$ and $Q_{13 B}$, with parallel base-emitter junctions and having the following saturation currents:

$$
\begin{aligned}
& I_{S A}=0.25 \times 10^{-14} \mathrm{~A} \\
& I_{S B}=0.75 \times 10^{-14} \mathrm{~A}
\end{aligned}
$$

Transistors $Q_{14}$ and $Q_{20}$ will be assumed to each have an area three times that of a standard device. Output transistors usually have relatively large areas, to be able to supply large load currents and dissipate relatively large amounts of power with only a moderate increase in device temperature.

## EXERCISE

x5.1 For the standard npn transistor whose parameters are given in Section x5.1 of this supplement, find approximate values for the following parameters at $I_{C}=0.1 \mathrm{~mA}: V_{B E}, g_{m}$, $r_{e}, r_{\pi}$, and $r_{o}$.

Ans. $575 \mathrm{mV} ; 4 \mathrm{~mA} / \mathrm{V} ; 250 \Omega ; 50 \mathrm{k} \Omega ; 1.25 \mathrm{M} \Omega$

## x5.1.2 DC Analysis

In this section, we shall carry out a dc analysis of the 741 circuit to determine the bias point of each device. For the dc analysis of an op-amp circuit, the input terminals are grounded. Theoretically speaking, this should result in zero dc voltage at the output. However, because the op amp has very large gain, any slight approximation in the analysis will show that the output voltage is far from being zero and is close to either $+V_{C C}$ or $-V_{E E}$. In actual practice, an op amp left open-loop will have an output voltage saturated close to one of the two supplies. To overcome this problem in the dc analysis, it will be assumed that the op amp is connected in a negative feedback loop that stabilizes the output dc voltage to zero volts.

Reference Bias Current The reference bias current $I_{\text {REF }}$ is generated in the branch composed of the two diode-connected transistors $Q_{11}$ and $Q_{12}$ and resistor $R_{5}$. Thus,

$$
I_{\mathrm{REF}}=\frac{V_{C C}-V_{E B 12}-V_{B E 11}-\left(-V_{E E}\right)}{R_{5}}
$$

For $V_{C C}=V_{E E}=15 \mathrm{~V}$ and $V_{B E 11}=V_{E B 12} \simeq 0.7 \mathrm{~V}$, we have $I_{\mathrm{REF}}=0.73 \mathrm{~mA}$.

Input-Stage Bias Transistors $Q_{11}$ and $Q_{10}$ and resistor $R_{4}$ form a Widlar current source (Section 8.7.4), thus

$$
\begin{equation*}
V_{T} \ln \frac{I_{\mathrm{REF}}}{I_{C 10}}=I_{C 10} R_{4} \tag{x5.1}
\end{equation*}
$$

## EXERCISE

x5.2 Use Eq. (x5.1) to determine the value of $I_{C 10}$ by trial and error. Note that $I_{\text {REF }}=0.73 \mathrm{~mA}$ and $R_{4}=5 \mathrm{k} \Omega$.

Ans. $I_{C 10}=19 \mu \mathrm{~A}$

Having determined $I_{C 10}$, we proceed to determine the dc current in each of the inputstage transistors. For this purpose, we show in Fig. x5.2 the centerpiece of the input stage: As will be seen shortly, this is a negative-feedback circuit that stabilizes the bias current of each of $Q_{1}$ to $Q_{4}$ at a value approximately equal to $I_{C 10} / 2$. Refer to the analysis indicated in the diagram (where $\beta_{N}$ is assumed to be high). The sum of the collector currents of $Q_{1}$ and $Q_{2}(2 I)$ is fed to (or sensed by) the input of the current mirror $Q_{8}-Q_{9}$. The output current of the mirror, which for large $\beta_{P}$ is approximately equal to $2 I$, is compared to $I_{C 10}$ at node X. The difference between the two currents $\left(2 I / \beta_{P}\right)$ establishes the base currents of $Q_{3}$ and $Q_{4}$. This is the error signal of the feedback loop. For large $\beta_{P}$, this current approaches zero and a node equation at X gives $2 I \simeq I_{C 10}$, and thus $I \simeq I_{C 10} / 2$.


Figure $x 5.2$ The dc analysis of the 741 input stage.

To verify the action of the negative-feedback loop in stabilizing the value of $I$, assume that for some reason $I$ increases. We see that the input current of the $Q_{8}-Q_{9}$ mirror increases and, correspondingly, its output current increases. Assuming that $I_{C 10}$ remains constant, consideration of node X reveals that the base currents in $Q_{3}$ and $Q_{4}$ must decrease. This in turn decreases the value of $I$, which is opposite to the originally assumed change.

## EXERCISES

x5.3 Using the value of $I_{C 10}$ found in Exercise x5.2, find the value of the bias current of each of $Q_{1}, Q_{2}, Q_{3}$, and $Q_{4}$.

Ans. $9.5 \mu \mathrm{~A}$
x5.4 Determine the loop gain of the feedback loop in Fig. x5.2. Break the loop at the input of the $Q_{8}-Q_{9}$ mirror. Since the input resistance of the mirror is low, ground the connection of the collectors of $Q_{1}$ and $Q_{2}$. Apply an input test current $i_{t}$ to the current mirror and find the feedback current that appears in the combined connection of the collectors of $Q_{1}$ and $Q_{2}$. Assume $I_{C 10}$ remains constant.

Ans. Loop gain $\simeq \beta_{P}$

Continuing with the dc analysis of the input stage, we show in Fig. x5.3 the currentmirror load $\left(Q_{5}, Q_{6}\right.$, and $\left.Q_{7}\right)$ and the input transistor of the second stage $\left(Q_{16}\right)$. The current-mirror load is fed by $I_{C 3}=I_{C 4} \simeq \mathrm{I}$. The analysis is illustrated in the figure and shows that for large $\beta_{N}$, each of $Q_{5}$ and $Q_{6}$ is biased at a current approximately equal to $I$. The bias current of $Q_{7}$ is somewhat higher, as shown in Exercise x5.5.


Figure $x 5.3$ Continuation of the dc analysis of the 741 input stage.

## EXERCISES

x5.5 Refer to Fig. x5.3 and recall that $I=9.5 \mu \mathrm{~A}, R_{1}=R_{2}=1 \mathrm{k} \Omega, R_{3}=50 \mathrm{k} \Omega, \beta_{N}=200$, and $I_{S}$ (for all three transistors) is $10^{-14} \mathrm{~A}$. Find $V_{B E 6}, V_{R 3}$, and $I_{C 7}$.

Ans. $517 \mathrm{mV} ; 526.5 \mathrm{mV} ; 10.5 \mu \mathrm{~A}$
x5.6 Recalling from Chapters 2 and 9 that the input bias current of an op amp is the average of its two input currents, thus

$$
I_{B}=\frac{1}{2}\left(I_{B 1}+I_{B 2}\right)
$$

and the input offset current is

$$
I_{O S}=\left|I_{B 1}-I_{B 2}\right|
$$

find $I_{B}$ and $I_{O S}$ for the 741 if $\beta_{1}$ and $\beta_{2}$ are nominally 200 but can deviate from nominal by as much as $\pm 5 \%$.

Ans. $47.5 \mathrm{nA} ; 4.75 \mathrm{nA}$

Input Common-Mode Range The input common-mode range is the range of input common-mode voltages over which the input stage remains in the linear active mode. Refer to Fig. x5.1. We see that in the 741 circuit the input common-mode range is determined at the upper end by saturation of $Q_{1}$ and $Q_{2}$, and at the lower end by saturation of $Q_{3}$ and $Q_{4}$.

## EXERCISE

x5.7 Neglect the voltage drops across $R_{1}$ and $R_{2}$ and assume that $V_{C C}=V_{E E}=15 \mathrm{~V}$. Show that the input common-mode range of the 741 is approximately -12.9 V to +14.7 V . (Assume that $V_{B E} \simeq 0.6 \mathrm{~V}$ and that to avoid saturation $V_{C B} \geq-0.3 \mathrm{~V}$ for an $n p n$ transistor, and $V_{B C} \geq-0.3$ V for a $p n p$ transistor.)

Second-Stage Bias If you refer to Fig. x5.1 you will see that if we neglect the base current of $Q_{23}$, the collector current of $Q_{17}$ will be equal to the current supplied by $Q_{13 B}$. We can then use $I_{C 17}$ to determine $V_{B E 17}, V_{B 17}$, the current through $R_{9}$ and hence $I_{E 16}$, and finally $I_{C 16} \simeq I_{E 16}$.

## EXERCISE

x5.8 Recalling that $Q_{13 B}$ has a scale current 0.75 times that of $Q_{12}$, find $I_{C 13 B}$ and hence $I_{C 17}$. Assume $\beta_{P} \gg 1$. Then determine $V_{B E 17}, I_{C 16}$, and $I_{B 16}$. (Recall that $I_{\mathrm{REF}}=0.73 \mathrm{~mA}, I_{S}=10-14$ A , and $\beta_{N}=200$.)

Ans. $550 \mu \mathrm{~A} ; 550 \mu \mathrm{~A} ; 618 \mathrm{mV} ; 16.2 \mu \mathrm{~A} ; 0.08 \mu \mathrm{~A}$

Output Stage Bias Figure x5.4 shows the output stage of the 741 with the short-circuit protection circuitry omitted. Current source $Q_{13 A}$ delivers a current of $0.25 I_{\text {REF }}$ (because $I_{S}$ of $Q_{13 A}$ is 0.25 times the $I_{S}$ of $Q_{12}$ ) to the network composed of $Q_{18}, Q_{19}$, and $R_{10}$. As mentioned in Section x5.1 of this supplement, the purpose of the $Q_{18}-Q_{19}$ network is to establish two $V_{B E}$ drops between the bases of the output transistors $Q_{14}$ and $Q_{20}$. If we neglect the base currents of $Q_{14}$ and $Q_{20}$, then the emitter current of $Q_{23}$ will also be equal to $0.25 I_{\text {REF }}$.

The determination of the bias currents of the output-stage transistors is illustrated by the following example.


Figure $\times 5.4$ The 741 output stage without the short-circuit-protection devices.

## Example x5.1

Determine $I_{C 23}, I_{B 23}, V_{B B}=V_{B E 18}+V_{B E 19}, I_{C 14}$, and $I_{C 20}$. Recall that $Q_{14}$ and $Q_{20}$ are nonstandard devices with $I_{S 14}=I_{S 20}=3 \times 10^{-14} \mathrm{~A}$.

## Solution

Fig. x5.4 shows that

$$
I_{C 23} \simeq I_{E 23} \simeq 0.25 I_{\mathrm{REF}}=180 \mu \mathrm{~A}
$$

Thus we see that the base current of $Q_{23}$ is only $180 / 50=3.6 \mu \mathrm{~A}$, which is negligible compared to $I_{C 17}$, as we assumed before.

If we assume that $V_{B E 18}$ is approximately 0.6 V , we can determine the current in $R_{10}$ as $15 \mu \mathrm{~A}$. The emitter current of $Q_{18}$ is therefore

$$
I_{E 18}=180-15=165 \mu \mathrm{~A}
$$

Also,

$$
I_{C 18} \simeq I_{E 18}=165 \mu \mathrm{~A}
$$

At this value of current we find that $V_{B E 18}=588 \mathrm{mV}$, which is quite close to the value assumed. The base current of $Q_{18}$ is $165 / 200=0.8 \mu \mathrm{~A}$, which can be added to the current in $R_{10}$ to determine the $Q_{19}$ current as

$$
I_{C 19} \simeq I_{E 19}=15.8 \mu \mathrm{~A}
$$

The voltage drop across the base-emitter junction of $Q_{19}$ can now be determined as

$$
V_{B E 19}=V_{T} \ln \frac{I_{C 19}}{I_{S}}=530 \mathrm{mV}
$$

The voltage drop $V_{B B}$ can now be calculated as

$$
V_{B B}=V_{B E 18}+V_{B E 19}=588+530=1.118 \mathrm{~V}
$$

Since $V_{B B}$ appears across the series combination of the base-emitter junctions of $Q_{14}$ and $Q_{20}$, we can write

$$
V_{B B}=V_{T} \ln \frac{I_{C 14}}{I_{S 14}}+V_{T} \ln \frac{I_{C 20}}{I_{S 20}}
$$

Using the calculated value of $V_{B B}$ and substituting $I_{S 14}=I_{S 20}=3 \times 10^{-14} \mathrm{~A}$, we determine the collector currents as

$$
I_{C 14}=I_{C 20}=154 \mu \mathrm{~A}
$$

This is the small current (relative to the load currents that the output stage is called upon to supply) at which the class $A B$ output stage is biased.

Summary For future reference, Table $\times 5.1$ provides a listing of the values of the collector bias currents of the 741 transistors.

| Table x5.1 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DC Collector Currents of the 741 Circuit $(\mu \mathrm{A})$ |  |  |  |  |  |  |  |
| $Q_{1}$ | 9.5 | $Q_{8}$ | 19 | $Q_{13 B}$ | 550 | $Q_{19}$ | 15.8 |
| $Q_{2}$ | 9.5 | $Q_{9}$ | 19 | $Q_{14}$ | 154 | $Q_{20}$ | 154 |
| $Q_{3}$ | 9.5 | $Q_{10}$ | 19 | $Q_{15}$ | 0 | $Q_{21}$ | 0 |
| $Q_{4}$ | 9.5 | $Q_{11}$ | 730 | $Q_{16}$ | 16.2 | $Q_{22}$ | 0 |
| $Q_{5}$ | 9.5 | $Q_{12}$ | 730 | $Q_{17}$ | 550 | $Q_{23}$ | 180 |
| $Q_{6}$ | 9.5 | $Q_{13 A}$ | 180 | $Q_{18}$ | 165 | $Q_{24}$ |  |
| $Q_{7}$ | 10.5 |  |  |  |  |  |  |

## EXERCISE

x5.9 If in the circuit of Fig. x5.4 the $Q_{18}-Q_{19}$ network is replaced by two diode-connected transistors, find the current in $Q_{14}$ and $Q_{20}$. Assume that the diode-connected transistors utilize standard devices with $I_{S}=10^{-14} \mathrm{~A}$, while the nonstandard $Q_{14}$ and $Q_{20}$ have $I_{S}=3 \times$ $10^{-14} \mathrm{~A}$.

Ans. $540 \mu \mathrm{~A}$

## x5.1.3 Small-Signal Analysis

The Input Stage Figure $x 5.5$ shows part of the 741 input stage for the purpose of performing small-signal analysis. Since the collectors of $Q_{1}$ and $Q_{2}$ are connected to a constant dc voltage, they are shown grounded. Also, the constant-current biasing of the bases of $Q_{3}$ and $Q_{4}$ is equivalent to having the common-base terminal open-circuited.

The differential signal $v_{i}$ applied between the input terminals effectively appears across four equal emitter resistances connected in series - those of $Q_{1}, Q_{2}, Q_{3}$, and $Q_{4}$. As a result, emitter signal currents flow as indicated in Fig. x5.5 with

$$
\begin{equation*}
i_{e}=\frac{v_{i}}{4 r_{e}} \tag{x5.2}
\end{equation*}
$$

where $r_{e}$ denotes the emitter resistance of each of $Q_{1}$ through $Q_{4}$. Thus

$$
r_{e}=\frac{V_{T}}{I}
$$

Thus the four transistors $Q_{1}$ through $Q_{4}$ supply the load circuit with a pair of complementary current signals $\alpha i_{e}$, as indicated in Fig. x5.5.

The input differential resistance of the op amp can be obtained from Fig. x5.5 as

$$
\begin{equation*}
R_{i d}=4\left(\beta_{N}+1\right) r_{e} \tag{x5.3}
\end{equation*}
$$

Proceeding with the input-stage analysis, we show in Fig. x5.6 the current-mirror-load circuit fed with the complementary pair of current signals found earlier. The analysis, together with the order of the steps in which it is performed, is indicated on the diagram. As expected, the current mirror provides an output current $i_{o}$,

$$
\begin{equation*}
i_{o}=2 \alpha i_{e} \tag{x5.4}
\end{equation*}
$$

Combining Eqs. (x5.2) and (x5.4) provides the transconductance of the input stage as

$$
\begin{equation*}
G_{m 1} \equiv \frac{i_{o}}{v_{i}}=\frac{\alpha}{2 r_{e}}=\frac{1}{2} g_{m 1} \tag{x5.5}
\end{equation*}
$$

where $g_{m 1}$ is the transconductance of each of the four transistors $Q_{1}, Q_{2}, Q_{3}$, and $Q_{4}$.


Figure x5.6 The current-mirror-load circuit of the input stage fed by the two complementary current signals generated by $Q_{1}$ through $Q_{4}$ in Fig. x5.5. Circled numbers indicate the order of the analysis steps.

## EXERCISES

x5.10 Recalling that each of the input-stage transistors is biased at a current $I=9.5 \mu \mathrm{~A}$ and that $\beta_{N}$ $=200$, find $r_{e}, g_{m 1}, G_{m 1}$, and $R_{i d}$.

Ans. $2.63 \mathrm{k} \Omega ; 0.38 \mathrm{~mA} / \mathrm{V} ; 0.19 \mathrm{~mA} / \mathrm{V} ; 2.1 \mathrm{M} \Omega$
x5.11 For the circuit in Fig. x5.6, find the following in terms of $i_{e}$ : (a) the signal voltage at the base of $Q_{6}$; (b) the signal current in the emitter of $Q_{7}$; (c) the signal current in the base of $Q_{7}$; (d) the signal voltage at the base of $Q_{7}$; (e) the input resistance seen by the left-hand-side signal current source $\alpha i_{e}$. Assume that $I_{C 7} \simeq I_{C 5}=I_{C 6}$, and use the results of Exercise $\times 5.10$.

Ans. (a) $3.63 \mathrm{k} \Omega \times i_{e}$; (b) $0.08 i_{e}$; (c) $0.0004 i_{e}$; (d) $3.84 \mathrm{k} \Omega \times i_{e}$; (e) $3.84 \mathrm{k} \Omega$

To complete our modeling of the 741 input stage, we must find its output resistance $R_{o 1}$. This is the resistance seen "looking back" into the output terminal of the circuit in Fig. x5.6. Thus $R_{o 1}$ is the parallel equivalent of the output resistance of the current source supplying the signal current $\alpha i_{e}$, and the output resistance of $Q_{6}$. The first component is the resistance looking into the collector of $Q_{4}$ in Fig. x5.5. Finding this resistance is considerably simplified if we assume that the common bases of $Q_{3}$ and $Q_{4}$ are at a virtual ground. This of course happens only when the input signal $\boldsymbol{v}_{i}$ is applied in a complementary fashion. Nevertheless, making this assumption does not result in a large error.

Assuming that the base of $Q_{4}$ is at virtual ground, the resistance we are after is $R_{o 4}$, shown in Fig. x5.7(a). This is the output resistance of a common-base transistor that has a resistance ( $r_{e}$ of $Q_{2}$ ) in its emitter. To find $R_{o 4}$ we use the following expression (Eq. 8.70):

$$
\begin{equation*}
R_{o}=r_{o}\left[1+g_{m}\left(R_{e} \| r_{\pi}\right)\right] \tag{x5.6}
\end{equation*}
$$

where $R_{e}=r_{e}$ and $r_{o}=V_{A p} / I$.
The second component of the output resistance is that seen looking into the collector of $Q_{6}$ in Fig. x 5.6 with the $\alpha i_{e}$ generator set to 0 . Although the base of $Q_{6}$ is not at signal ground, we shall assume that the signal voltage at the base is small enough to make this approximation valid. The circuit then takes the form shown in Fig. x5.7(b), and $R_{o 6}$ can be determined using Eq. (x5.6) with $R_{e}=R_{2}$.

Figure $x 5.8$ shows the equivalent circuit that we have derived for the input stage.


Figure x5.7 Simplified circuits for finding the two components of the output resistance $R_{o_{1}}$ of the first stage.


Figure x5.8 Small-signal equivalent circuit for the input stage of the 741 op amp .

## EXERCISES

x5.12 Find the values of $R_{o 4}$ and $R_{o 6}$ and thus the output resistance of the first stage, $R_{o 1}$. Recall that $I=9.5 \mu \mathrm{~A}, V_{a n}=125 \mathrm{~V}, V_{A p}=50 \mathrm{~V}, R_{2}=1 \mathrm{k} \Omega, \beta_{N}=200$, and $\beta_{P}=50$.

Ans. 10.5 M $\Omega$; 18.2 M $\Omega ; 6.7 \mathrm{M} \Omega$
x5.13 Use the equivalent circuit of Fig. x5.8 together with the value of $G_{m 1}$ found in Exercise x5.10 and the value of $R_{o^{1}}$ found in Exercise x5.12 to determine the open-circuit voltage gain of the 741 input stage.

Ans. $\left|A_{v_{o}}\right|=G_{m 1} R_{o 1}=1273 \mathrm{~V} / \mathrm{V}$

## Example x5.2

We wish to find the input offset voltage resulting from a $2 \%$ mismatch between the resistances $R_{1}$ and $R_{2}$ in Fig. x5.1.

## Solution

Consider first the situation when both input terminals are grounded, and assume that $R_{1}=R$ and $R_{2}$ $=R+\Delta R$, where $\Delta R / R=0.02$. From Fig. x5.9 we see that while Q 5 still conducts a current equal to I, the current in Q6 will be smaller by $\Delta I$. The value of $\Delta I$ can be found from

$$
V_{B E 5}+I_{R}=V_{B E 6}+(I-\Delta I)(R+\Delta R)
$$

Thus,

$$
\begin{equation*}
V_{B E 5}-V_{B E 6}=I \Delta R-\Delta I(R+\Delta R) \tag{x5.7}
\end{equation*}
$$

The quantity on the left-hand side is in effect the change in $V_{B E}$ due to a change in $I_{E}$ of $\Delta I$. We may therefore write

$$
\begin{equation*}
V_{B E 5}-V_{B E 6} \simeq \Delta I r_{e} \tag{x5.8}
\end{equation*}
$$

Equations (x5.7) and (x5.8) can be combined to obtain

$$
\begin{equation*}
\frac{\Delta I}{I}=\frac{\Delta R}{R+\Delta R+r_{e}} \tag{x5.9}
\end{equation*}
$$

Substituting $R=1 \mathrm{k} \Omega$ and $r_{e}=2.63 \mathrm{k} \Omega$ shows that a $2 \%$ mismatch between $R_{1}$ and $R_{2}$ gives rise to an output current $\Delta I=5.5 \times 10^{-3} I$. To reduce this output current to zero we have to apply an input voltage $V_{O S}$ given by

$$
\begin{equation*}
V_{O S}=\frac{\Delta I}{G_{m 1}}=\frac{5.5 \times 10^{-3} I}{G_{m 1}} \tag{x5.10}
\end{equation*}
$$

Substituting $I=9.5 \mu \mathrm{~A}$ and $G_{m 1}=0.19 \mathrm{~mA} / \mathrm{V}$ results in the offset voltage $V_{O S} \simeq 0.3 \mathrm{mV}$.
It should be pointed out that the offset voltage calculated is only one component of the input offset voltage of the 741. Other components arise because of mismatches in transistor characteristics. The 741 offset voltage is specified to be typically 2 mV .


Figure x5.9 Input stage with both inputs grounded and a mismatch $\Delta R$ between $R_{1}$ and $R_{2}$.

## Example x5.3

We are required to find the CMRR of the 741 input stage. Assume that the circuit is balanced except for mismatches in the current-mirror load that result in an error $\epsilon_{m}$ in the mirror's currenttransfer ratio; that is, the ratio becomes $\left(1-\epsilon_{m}\right)$.

## Solution

In Section 9.5 .5 we analyzed the common-mode operation of the current-mirror-loaded differential amplifier and derived an expression for its CMRR. The situation in the 741 input stage, however, differs substantially because of the feedback loop that regulates the bias current. Since this feedback loop is sensitive to the common-mode signal, as will be seen shortly, the loop operates to reduce the common-mode gain and, correspondingly, to increase the CMRR. Hence, its action is referred to as common-mode feedback.

Figure x 5.10 shows the 741 input stage with a common-mode signal $\boldsymbol{v}_{\text {icm }}$ applied to both input terminals. We have assumed that as a result of $\boldsymbol{v}_{i c m}$, a signal current $i$ flows as shown. Since the stage is balanced, both sides carry the same current $i$.

Our objective now is to determine how $i$ relates to $v_{i c m}$. Toward that end, observe that for common-mode inputs, both sides of the differential amplifier, that is, $Q_{1}-Q_{3}$ and $Q_{2}-Q_{4}$, act as
followers, delivering a signal almost equal to $\boldsymbol{v}_{\text {icm }}$ to the common-base node of $Q_{3}$ and $Q_{4}$. Now, this node Y is connected to the collectors of two current sources, $Q_{9}$ and $Q_{10}$. Denoting the total resistance between node Y and ground $R_{o}$, we write

$$
\begin{equation*}
R_{o}=R_{o 9} \| R_{o 10} \tag{x5.11}
\end{equation*}
$$

In Fig. x 5.10 we have "pulled $R_{o}$ out," thus leaving behind ideal current sources $Q_{9}$ and $Q_{10}$. Since the current in $Q_{10}$ is constant, we show $Q_{10}$ in Fig. x5.10 as having a zero incremental current. Transistor $Q_{9}$, on the other hand, provides a current approximately equal to that fed into $Q_{8}$, which is $2 i$. This is the feedback current. Since $Q_{8}$ senses the sum of the currents in the two sides of the differential amplifier, the feedback loop operates only on the common-mode signal and is insensitive to any difference signal.

Proceeding with the analysis, we now can write a node equation at Y ,

$$
\begin{equation*}
2 i+\frac{2 i}{\beta_{P}}=\frac{v_{i c m}}{R_{o}} \tag{x5.12}
\end{equation*}
$$

Assuming $\beta_{P} \gg 1$, this equation simplifies to

$$
\begin{equation*}
i \simeq \frac{v_{i c m}}{s R_{o}} \tag{x5.13}
\end{equation*}
$$

Having determined $i$, we now proceed to complete our analysis by finding the output current $i_{o}$. From the circuit in Fig. x5.10, we see that

$$
\begin{equation*}
i_{o}=\epsilon_{m} i \tag{x5.14}
\end{equation*}
$$

Thus the common-mode transconductance of the input stage is given by

$$
G_{m c m} \equiv \frac{i_{o}}{v_{i c m}}=\frac{\epsilon_{m} i}{v_{i c m}}
$$

Substituting for $i$ from Eq. (x5.13) gives

$$
\begin{equation*}
G_{m c m}=\frac{\epsilon_{m}}{2 R_{o}} \tag{x5.15}
\end{equation*}
$$

Finally, the CMRR can be found as the ratio of the differential transconductance $G_{m 1}$ found in Eq. (x5.5) and the common-mode transconductance $G_{m c m}$,

$$
\begin{equation*}
\mathrm{CMRR} \equiv \frac{G_{m 1}}{G_{m c m}}=g_{m 1} R_{o} / \epsilon_{m} \tag{x5.16}
\end{equation*}
$$

where $g_{m 1}$ is the transconductance of $Q_{1}$. Now substituting for $R_{o}$ from Eq. (x5.11), we obtain

$$
\begin{equation*}
\operatorname{CMRR}=g_{m 1}\left(R_{o 9} \| R_{o 10}\right) / \epsilon_{m} \tag{x5.17}
\end{equation*}
$$

Before leaving this example, we observe that if the feedback were not present, the 2 i term in Eq. (x5.12) would be absent and the current i would become $\beta_{P}\left(\boldsymbol{v}_{i c m} / 2 R_{o}\right)$, which is $\beta_{P}$ times higher than that when feedback is present. In other words, common-mode feedback reduces i, hence the common-mode transconductance and the common-mode gain, by a factor $\beta_{P}$. It can be shown that $\beta_{P}$ is the magnitude of the loop gain. (See Exercise x5.4.)


Figure x5.10 Example x5.3: Analysis of the common-mode gain of the 741 input stage. Note that $R_{o}=R_{o 9} \| R_{o 10}$ has been "pulled out" and shown separately, leaving behind ideal current sources $Q_{9}$ and $Q_{10}$.

## EXERCISES

x5.14 Show that if the source of the imbalance in the current-mirror load is that while $R_{1}=R, R_{2}=$ $R+\Delta R$, the error $\epsilon_{m}$ is given by

$$
\epsilon_{m}=\frac{\Delta R}{R+r_{e S}+\Delta R}
$$

Evaluate $\epsilon_{m}$ for $\Delta R / R=0.02$.
Ans. $\epsilon_{m}=5.5 \times 10^{-3}$
x5.15 Refer to Fig. x5.10 and assume that the bases of $Q_{9}$ and $Q_{10}$ are at approximately constant voltages (signal ground). Find $R_{o 9}, R_{o 10}$, and hence $R_{o}$. Use $V_{A}=125 \mathrm{~V}$ for $n p n$ and 50 V for $p n p$ transistors. Use the bias current values in Table 13.1.

Ans. $R_{o 9}=2.63 \mathrm{M} \Omega ; R_{o 10}=31.1 \mathrm{M} \Omega ; R_{o}=2.43 \mathrm{M} \Omega$
x5.16 Use the results of Exercises x5.14 and x5.15 to determine $G_{m c m}$ and CMRR of the 741 input stage. What would the CMRR be if the common-mode feedback were not present? Assume $\beta_{P}=50$.

Ans. $G_{m c m}=1.13 \times 10^{-6} \mathrm{~mA} / \mathrm{V} ; \mathrm{CMRR}=1.68 \times 10^{5}$ or 104.5 dB ; without common-mode feedback, $\mathrm{CMRR}=70.5 \mathrm{~dB}$

The Second Stage Figure $x 5.11$ (a) shows the 741 second stage prepared for small-signal analysis, and Fig. x5.11(b) shows its small-signal model. The three model parameters $R_{i 2}$, $G_{m 2}$, and $R_{o 2}$ can be determined as follows.

The input resistance $R_{i 2}$ can be found by inspection to be

$$
\begin{equation*}
R_{R 2}=\left(\beta_{16}+1\right)\left\{r_{e 16}+\left[R_{9} \|\left(\beta_{17}+1\right)\left(r_{e 17}+R_{8}\right)\right]\right\} \tag{x5.18}
\end{equation*}
$$

From the equivalent circuit of Fig. x5.11(b), we see that the transconductance $G_{m 2}$ is the ratio of the short-circuit output current to the input voltage. Short-circuiting the output terminal of the second stage (Fig. x5.11a) to ground makes the signal current through the output resistance of $Q_{13 B}$ zero, and the output short-circuit current becomes equal to the collector signal current of $Q_{17}\left(i_{c 17}\right)$. This latter current can be easily related to $v_{i 2}$ as follows:

$$
\begin{equation*}
i_{c 17}=\frac{\alpha v_{b 17}}{r_{e 17}+R_{8}} \tag{x5.19}
\end{equation*}
$$



Figure $x 5.11$ (a) The 741 second stage prepared for small-signal analysis. (b) Equivalent circuit.

$$
\begin{align*}
& v_{b 17}=v_{i 2} \frac{\left(R_{g} \| R_{i 17}\right)}{\left(R_{9} \| R_{i 17}\right)+r_{e 16}}  \tag{x5.20}\\
& R_{i 17}=\left(\beta_{17}+1\right)\left(r_{e 17}+R_{8}\right) \tag{x5.21}
\end{align*}
$$

where we have neglected $r_{o 16}$ because $r_{o 16} \gg R_{9}$. These equations can be combined to obtain

$$
\begin{equation*}
G_{m 2} \equiv \frac{i_{c 17}}{v_{i 2}} \tag{x5.22}
\end{equation*}
$$

To determine the output resistance $R_{o 2}$ of the second stage in Fig. x5.11(a), we ground the input terminal and find the resistance looking back into the output terminal. It follows that $R_{o 2}$ is given by

$$
\begin{equation*}
R_{o 2}=\left(R_{o 13 B} \| R_{o 17}\right) \tag{x5.23}
\end{equation*}
$$

where $R_{o 13 B}$ is the resistance looking into the collector of $Q_{13 B}$ while its base and emitter are connected to ground. It can be easily seen that

$$
\begin{equation*}
R_{o 13 B}=r_{o 13 B} \tag{x5.24}
\end{equation*}
$$

The second component in Eq. (x5.23), $R_{o 17}$, is the resistance seen looking into the collector of $Q_{17}$. Since the resistance between the base of $Q_{17}$ and ground is relatively small (approximately equal to $r_{e 16}$ ), one can considerably simplify matters by assuming that the base is grounded. Doing this, we can use Eq. (x5.6) to determine $R_{o 17}$.

## EXERCISES

In the following exercises use $I_{C 13 B}=550 \mu \mathrm{~A}, I_{C 16}=16.2 \mu \mathrm{~A}, I_{C 17}=550 \mu \mathrm{~A}, \beta_{N}=200, \beta_{P}=$ $50, V_{A n}=125 \mathrm{~V}, V_{A p}=50 \mathrm{~V}, R_{9}=50 \mathrm{k} \Omega$, and $R_{8}=100 \Omega$.
x5.17 Determine the value of $R_{i 2}$.

Ans. $4 \mathrm{M} \Omega$
x5.18 Determine the value of $G_{m 2}$.

Ans. $6.5 \mathrm{~mA} / \mathrm{V}$
x5.19 Determine the values of $R_{o 13 B}, R_{o 17}$, and $R_{o 2}$.

Ans. $90.9 \mathrm{k} \Omega ; 722 \mathrm{k} \Omega ; 81 \mathrm{k} \Omega$
$x 5.20$ Determine the value of the open-circuit voltage gain of the second stage.
Ans. $-526.5 \mathrm{~V} / \mathrm{V}$

The Output Stage The 741 output stage is shown in Fig. x 5.12 without the short-circuit protection circuitry. The stage is shown driven by the second-stage transistor $Q_{17}$ and loaded with a $2-\mathrm{k} \Omega$ resistance. The circuit is of the AB class (Section 12.4), with the network composed of $Q_{18}, Q_{19}$, and $R_{10}$ providing the bias of the output transistors $Q_{14}$ and $Q_{20}$. The use of this network rather than two diode-connected transistors in series enables biasing the output transistors at a low current $(0.15 \mathrm{~mA})$ in spite of the fact that the output devices are three times as large as the standard devices. This result is obtained by arranging that the current in $Q_{19}$ is very small and thus its $V_{B E}$ is also small. We analyzed the dc bias in Section x5.2 of the bonus material.

Another feature of the 741 output stage worth noting is that the stage is driven by an emitter follower $Q_{23}$. As will be shown, this emitter follower provides added buffering, which makes the op-amp gain almost independent of the parameters of the output transistors.

Let's first determine the allowable range of output voltage swing. The maximum positive output voltage is limited by the saturation of current-source transistor $Q_{13 A}$. Thus,

$$
\begin{equation*}
v_{O \max }=V_{C C}-\left|V_{C E s a t}\right|-V_{B E 14} \tag{x5.25}
\end{equation*}
$$

which is about 1 V below $V_{C C}$. The minimum output voltage (i.e., maximum negative amplitude) is limited by the saturation of $Q_{17}$.


Figure $\times 5.12$ The 741 output stage without the short-circuit-protection circuitry.

Neglecting the voltage drop across $R_{8}$, we obtain

$$
\begin{equation*}
v_{O \min }=-V_{E E}+V_{C E s a t}+V_{E B 23}+V_{E B 20} \tag{x5.26}
\end{equation*}
$$

which is about 1.5 V above $-V_{E E}$.
Next, we consider the small-signal analysis of the output stage. Specifically, we model the output stage using the equivalent circuit in Fig. x5.13 and determine the model parameters as follows. Note that the model is shown fed with the open-circuit voltage of the second stage $v_{o 2}$, where from Fig. x5.11(b), $v_{o 2}=-G_{m 2} R_{o 2} v_{i 2}$.

To determine the input resistance $R_{\text {in3 }}$, we take into account the load resistance $R_{L}$ and assume that one of the output transistors is conducting, as shown in the following example.


Figure $\times 5.13$ Model for the 741 output stage in Fig. x5.12.

## Example x5.4

Assuming that $Q_{14}$ is off and $Q_{20}$ is conducting a current of 5 mA to a load $R_{L}=2 \mathrm{k} \Omega$, determine the value of $R_{\mathrm{in} 3}$. Using $G_{m 2}=6.5 \mathrm{~mA} / \mathrm{V}$ and $R_{o 2}=81 \mathrm{k} \Omega$, determine the voltage gain of the second stage.

## Solution

Refer to Fig. x5.12. The input resistance looking into the base of $Q_{20}$ is approximately $\beta_{20} R_{L}=50$ $\times 2=100 \mathrm{k} \Omega$. This resistance appears in parallel with the series combination of $r_{o 13 A}=V_{A p} / I_{C 13 A}=$ $50 \mathrm{~V} / 180 \mu \mathrm{~A}=280 \mathrm{k} \Omega$, and the resistance of the $Q_{18}-Q_{19}$ network. The latter resistance is very small (about $160 \Omega$; see later: Exercise x5.22). Thus, the total resistance in the emitter of $Q_{23}$ is approximately $(100 \mathrm{k} \Omega \| 280 \mathrm{k} \Omega)$ or $74 \mathrm{k} \Omega$, and the input resistance $R_{\text {in } 3}$ is obtained as

$$
R_{\mathrm{in} 3}=\beta_{23} \times 74 \mathrm{k} \Omega=50 \times 74=3.7 \mathrm{M} \Omega
$$

We thus see that $R_{\text {in } 3} \gg R_{o 2}$, and the value of $R_{\text {in } 3}$ will have little effect on the performance of the op amp. Still we can determine the gain of the second stage as

$$
\begin{aligned}
A_{2} & \equiv \frac{v_{i 3}}{v_{i 2}}=-G_{m 2} R_{o 2} \frac{R_{\mathrm{in} 3}}{R_{\mathrm{in} 3}+R_{o 2}} \\
& =-6.5 \times 81 \frac{3700}{3700+81}=-515 \mathrm{~V} / \mathrm{V}
\end{aligned}
$$

Continuing with the determination of the equivalent-circuit model parameters, we note from Fig. x5.13 that $G_{v o 3}$ is the open-circuit overall voltage gain of the output stage,

$$
\begin{equation*}
G_{v o 3}=\left.\frac{v_{o}}{v_{o 2}}\right|_{R_{L}=\infty} \tag{x5.27}
\end{equation*}
$$

With $R_{L}=\infty$, the gain of the emitter-follower output transistor ( $Q_{14}$ or $Q_{20}$ ) will be nearly unity. Also, with $R_{L}=\infty$ the resistance in the emitter of $Q_{23}$ will be very large. This means that the gain of $Q_{23}$ will be nearly unity and the input resistance of $Q_{23}$ will be very large. We thus conclude that $G_{v o 3} \simeq 1$.

Next, we shall find the value of the output resistance of the op amp, $R_{\text {out }}$. For this purpose, refer to the circuit shown in Fig. x5.14. In accordance with the definition of $R_{\text {out }}$ from Fig. x5.13, the input source feeding the output stage is grounded, but its resistance (which is the output resistance of the second stage, $R_{o 2}$ ) is included. We have assumed that the output voltage $v_{O}$ is negative, and thus $Q_{20}$ is conducting most of the current; transistor $Q_{14}$ has therefore been eliminated. The exact value of the output resistance will of course depend on which transistor ( $Q_{14}$ or $Q_{20}$ ) is conducting and on the value of load current. Nevertheless, we wish to find an estimate of $R_{\text {out }}$. The analysis for doing so is shown in Fig. x5.14. It should be noted, however, that to the value of $R_{\text {out }}$ given in the figure we must add the resistance $R_{7}(27 \Omega)$ (see Fig. x5.1), which is included for shortcircuit protection, in order to obtain the total output resistance of the 741.


Figure x5.14 Circuit for finding the output resistance $R_{\text {out }}$.

## EXERCISES

x5.21 Find the value of $R_{o 23}, R_{\text {out }}$, and the total output resistance of the 741 op amp . Use $R_{o 2}=81$ $\mathrm{k} \Omega, \beta_{23}=\beta_{20}=50$, and $I_{C 23}=180 \mu \mathrm{~A}$, and assume that $Q_{20}$ is conducting a load current of 5 mA .

Ans. $1.73 \mathrm{k} \Omega ; 39 \Omega ; 66 \Omega$
x5.22 Using a simple $\left(r_{\pi}, g_{m}\right)$ model for each of the two transistors $Q_{18}$ and $Q_{19}$ in Fig. x5.15, find the small-signal resistance between $A$ and $A^{\prime}$. (Note: From Table x5.1, $I_{C 18}=165 \mu \mathrm{~A}$ and $I_{C 19}$ $\simeq 16 \mu \mathrm{~A}$. Also, $\beta_{N}=200$.)


$$
r_{A A^{\prime}} \equiv \frac{v_{t}}{i}
$$

Figure $\times 5.15$
Ans. $163 \Omega$

Output Short-Circuit Protection If the op-amp output terminal is short-circuited to one of the power supplies, one of the two output transistors could conduct a large amount of current. Such a large current can result in sufficient heating to cause burnout of the IC (Chapter 12). To guard against this possibility, the 741 op amp is equipped with a special circuit for short-circuit protection. The function of this circuit is to limit the current in the output transistors in the event of a short circuit.

Refer to Fig. x5.1 and note that the short-circuit-protection circuitry is highlighted in color. Resistance $R_{6}$ together with transistor $Q_{15}$ limits the current that would flow out of $Q_{14}$ in the event of a short circuit. Specifically, if the current in the emitter of $Q_{14}$ exceeds about 20 mA , the voltage drop across $R_{6}$ exceeds 540 mV , which turns $Q_{15}$ on. As $Q_{15}$ turns on, its collector robs some of the current supplied by $Q_{13 A}$, thus reducing the base current of $Q_{14}$. This mechanism thus limits the maximum current that the op amp can source (i.e., supply from the output terminal in the outward direction) to about 20 mA .

Limiting of the maximum current that the op amp can sink, and hence the current through $Q_{20}$, is done by a mechanism similar to the one discussed above. The relevant circuit is composed of $R_{7}, Q_{21}, Q_{24}$, and $Q_{22}$. For the components shown, the current in the inward direction is limited also to about 20 mA .

Overall Voltage Gain The overall small-signal gain can be found from the cascade of the equivalent circuits derived above for the three op-amp stages. This cascade is shown in Fig. x5.16, loaded with $R_{L}=2 \mathrm{k} \Omega$, which is the typical value used in measuring and specifying the 741 data. The overall gain can be expressed as

$$
\begin{align*}
\frac{v_{o}}{v_{i}} & =\frac{v_{i 2}}{v_{i}} \frac{v_{o 2}}{v_{i 2}} \frac{v_{o}}{v_{o 2}} \\
& =-G_{m 1}\left(R_{o 1} \| R_{i 2}\right)\left(-G_{m 2} R_{o 2}\right) G_{v o 3} \frac{R_{L}}{R_{L}+R_{\text {out }}} \tag{x5.28}
\end{align*}
$$

Using the values found earlier yields for the overall open-circuit voltage gain,

$$
\begin{align*}
A_{0} & \equiv \frac{v_{o}}{v_{i}}=-476.1 \times(-526.5) \times 0.97=243,147 \frac{\mathrm{~V}}{\mathrm{~V}} \\
& =107.7 \mathrm{~dB} \tag{x5.29}
\end{align*}
$$

## x5.1.4 Frequency Response

The 741 is an internally compensated op amp. It employs the Miller compensation technique, studied in Section 11.10.3, to introduce a dominant low-frequency pole. Specifically, a $30-\mathrm{pF}$ capacitor $\left(C_{C}\right)$ is connected in the negative-feedback path of the second stage. An approximate estimate of the frequency of the dominant pole can be obtained as follows.

From Miller's theorem (Section 10.2.5), we see that the effective capacitance due to $C_{C}$ between the base of $Q_{16}$ and ground is (see Fig. x5.1)

$$
\begin{equation*}
C_{\mathrm{in}}=C_{C}\left(1+\left|A_{2}\right|\right) \tag{x5.30}
\end{equation*}
$$

where $A_{2}$ is the second-stage gain. Use of the value calculated for $A_{2}$ found in Example x5.4, $A_{2}=-515$, results in $C_{\text {in }}=15,480 \mathrm{pF}$. Since this capacitance is quite large, we shall neglect all other capacitances between the base of $Q_{16}$ and signal ground. The total resistance between this node and ground is


Figure x5.16 Cascading the small-signal equivalent circuits of the individual stages for the evaluation of the overall voltage gain.

$$
\begin{align*}
R_{t} & =R_{o 1} \| R_{i 2} \\
& =6.7 \mathrm{M} \Omega \| 4 \mathrm{M} \Omega=2.5 \mathrm{M} \Omega \tag{x5.31}
\end{align*}
$$

Thus the dominant pole has a frequency $f_{P}$ given by

$$
\begin{equation*}
f_{P}=\frac{1}{2 \pi C_{\mathrm{in}} R_{t}}=4.1 \mathrm{~Hz} \tag{x5.32}
\end{equation*}
$$

Note that this approach is equivalent to using the appropriate formula in Eq. (11.58) found in the textbook.

As discussed in Section 11.10.3, Miller compensation provides an additional advantageous effect, namely, pole splitting. As a result, the other poles of the circuit are moved to very high frequencies. This has been confirmed by computer-aided analysis (see Gray et al., 2000).

Assuming that all nondominant poles are at very high frequencies, the calculated values give rise to the Bode plot shown in Fig. x5.17, where $f_{3 \mathrm{~dB}}=f_{P}$. The unity-gain bandwidth $f_{t}$ can be calculated from

$$
\begin{equation*}
f_{t}=A_{0} f_{3 \mathrm{~dB}} \tag{x5.33}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
f_{t}=243,147 \times 4.1 \simeq 1 \mathrm{MHz} \tag{x5.34}
\end{equation*}
$$

Although this Bode plot implies that the phase shift at $f_{t}$ is $-90^{\circ}$ and thus that the phase margin is $90^{\circ}$, in practice a phase margin of about $80^{\circ}$ is obtained. The excess phase shift (about $10^{\circ}$ ) is due to the nondominant poles. This phase margin is sufficient to provide stable operation of closed-loop amplifiers with any value of feedback factor $\beta$.


Figure $\times 5.17$ Bode plot for the 741 gain, neglecting nondominant poles.

This convenience of use of the internally compensated 741 is achieved at the expense of a great reduction in open-loop gain and hence in the amount of negative feedback. In other words, if one requires a closed-loop amplifier with a gain of 1000 , then the 741 is overcompensated for such an application, and one would be much better off designing one's own compensation (assuming, of course, the availability of an op amp that is not already internally compensated).

A Simplified Model The simplified model of the 741 op amp shown in Fig. x5.18 is similar to what we used for the CMOS two-stage op amp (Section 13.1.5). Here, however, the high-gain second stage, with its feedback capacitance $C_{C}$, is modeled by an ideal integrator. In this model, the gain of the second stage is assumed to be sufficiently large that a virtual ground appears at its input. For this reason the output resistance of the input stage and the input resistance of the second stage have been omitted. Furthermore, the output stage is assumed to be an ideal unity-gain follower. (Of course, the two-stage CMOS amplifier does not have an output stage.)

Analysis of the model in Fig. x5.18 gives

$$
\begin{equation*}
A(s) \equiv \frac{V_{o}(s)}{V_{i}(s)}=\frac{G_{m 1}}{s C_{C}} \tag{x5.35}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
A(j \omega)=\frac{G_{m 1}}{j \omega C_{C}} \tag{x5.36}
\end{equation*}
$$

Substituting $\mathrm{Gml}-0.19 \mathrm{~mA} / \mathrm{V}$ and $\mathrm{CC}=30 \mathrm{pF}$ yields

$$
\begin{equation*}
f_{t}=\frac{\omega_{t}}{2 \pi} \simeq 1 \mathrm{MHz} \tag{x5.37}
\end{equation*}
$$

which is equal to the value calculated before. It should be pointed out, however, that this model is valid only at frequencies $f \gg f_{3 \mathrm{~dB}}$. At such frequencies, the gain falls off with a slope of $-20 \mathrm{~dB} /$ decade (Fig. x5.17), just like that of an integrator.


Figure x 5.18 A simple model for the 741 based on modeling the second stage as an integrator.

## x5.1.5 Slew Rate

The slew-rate limitation of op amps is discussed in Chapter 2, and expressions for $S R$ are derived for the two-stage CMOS op amp in Section 13.1 and for the folded-cascode CMOS op amp in Section 13.2. The 741 slewing is very similar to that of the two-stage CMOS op amp. Thus, following an identical procedure, we can show that for the 741 op amp,

$$
\begin{equation*}
S R=\frac{2 I}{C_{C}} \tag{x5.38}
\end{equation*}
$$

where $2 I$ is the total bias current of the input differential stage. For the $741, I=9.5 \mu \mathrm{~A}$, and $C_{C}=30 \mathrm{pF}$, resulting in $S R=0.63 \mathrm{~V} / \mu \mathrm{s}$.

Also, as we have done for the two-stage CMOS op amp, we can derive a relationship between $S R$ and $\omega_{t}$. For the 741 case, we can show that

$$
\begin{equation*}
S R=4 V_{T} \omega_{t} \tag{x5.39}
\end{equation*}
$$

where $V_{T}$ is the thermal voltage (approximately 25 mV at room temperature). As a check, for the 741 we have

$$
S R=4 \times 25 \times 10^{-3} \times 2 \pi \times 10^{6}=0.63 \mathrm{~V} / \mu \mathrm{s}
$$

which is the result obtained previously. Observe that Eq. (x5.49) is of the same form as Eq. (13.42), which applies to the two-stage CMOS op amp. Here, $4 V_{T}$ replaces $V_{O V}$. Since, typically, $V_{O V}$ will be two to three times the value of $4 V_{T}$, a two-stage CMOS op amp with an $f_{t}$ equal to that of the 741 exhibits a slew rate that is two to three times as large as that of the 741.

## EXERCISE

x5.23 Use the value of the slew rate calculated above to find the full-power bandwidth $f_{M}$ of the 741 op amp. Assume that the maximum output is $\pm 10 \mathrm{~V}$.

Ans. 10 kHz

## x5.2 Special Low-Supply-Voltage Performance Requirements

Many special performance requirements stem from the need to operate modern op amps from power supplies of much lower voltages. Thus while the 741-type op amp operated from $\pm 15-\mathrm{V}$ power supplies, many modern BJT op amps are required to operate from a single power supply of only 2 V to 3 V . This is done for a number of reasons, including the following:

1. Modern small-feature-size IC fabrication technologies require low power-supply voltages.
2. Compatibility must be achieved with other parts of the system that use lowvoltage supplies.
3. Power dissipation must be minimized, especially for battery-operated equipment.

As Fig. x5.19 indicates, there are two important changes: the use of a single groundreferenced power supply $V_{C C}$, and the low value of $V_{C C}$. Both of these requirements give rise to changes in performance specifications and pose new design challenges. In the following we discuss two of the resulting changes.


Figure x5.19 Power-supply requirements have changed considerably. Modern BJT op amps are required to operate from a single supply $V_{C c}$ of 2 V to 3 V .

## x5.2.1 Rail-to-Rail Input Common-Mode Range

Recall that the input common-mode range of an op amp is the range of common-mode input voltages for which the op amp operates properly and meets its performance specifications, such as voltage gain and CMRR. Op amps of the 741 type operate from $\pm 15-\mathrm{V}$ supplies and exhibit an input common-mode range that extends to within a couple of volts of each supply. Such a gap between the input common-mode range and the power supply is obviously unacceptable if the op amp is to be operated from a single supply that is only 2 V to 3 V . Indeed we will now show that these single-supply, lowvoltage op amps need to have an input common-mode range that extends over the entire supply voltage, 0 to $V_{C C}$, referred to as rail-to-rail input common-mode range.

Consider first the inverting op-amp configuration shown in Fig. x5.20(a). Since the positive input terminal is connected to ground (which is the voltage of the negative supply rail), ground voltage has to be within the allowable input common-mode range. In fact, because for positive output voltages the voltage at the inverting input terminal can go slightly negative, the input common-mode range should extend below the negative supply rail (ground).

Next consider the unity-gain voltage follower obtained by applying $100 \%$ negative feedback to an op amp, as shown in Fig. x5.20(b). Here the input common-mode voltage is equal to the input signal $v_{I}$. To maximize the usefulness of this buffer amplifier, its input signal $\boldsymbol{v}_{I}$ should be allowed to extend from 0 to $V_{C C}$, especially since $V_{C C}$ is only 2 V to 3 V . Thus the input common-mode range should include also the positive supply rail. As shown in Section 13.3.2, modern BJT op amps can operate over an input commonmode voltage range that extends a fraction of a volt beyond its two supply rails: that is, more than rail-to-rail operation!


Figure $x 5.20$ (a) In the inverting configuration, the positive op-amp input is connected to ground; thus it is imperative that the input common-mode range includes ground voltage. (b) In the unity-gain follower configuration, $v_{I C M}=v_{I}$; thus it is highly desirable for the input common-mode range to include ground voltage and $V_{C C}$.

## x5.2.2 Near Rail-to-Rail Output Signal Swing

In the 741 op amp, we were satisfied with an output that can swing to within 2 Vor so of each of the supply rails. With a supply of $\pm 15 \mathrm{~V}$, this capacity resulted in a respectable $\pm 13-\mathrm{V}$ output range. However, to limit the output swing to within 2 V of the supply rails in an op amp operating from a single $3-\mathrm{V}$ supply would result in an unable device! Thus, here too, we require near rail-to-rail operation. As shown in Section 13.3.5, this requirement forces us to adopt a whole new approach to output-stage design.

